LINEAR CIRCUIT ANALYSIS (EED) – U.E.T. TAXILA | 11

ENGR. M. MANSOOR ASHRAF

INTRODUCTION

Power analysis is of paramount importance.

Power is the most important quantity in electric utilities, communication, electronic and power systems.

Every industrial and household electrical device has a power rating that indicates how much power the equipment requires.

Generally there are three types of electrical power: Active power, Reactive power and Apparent power.

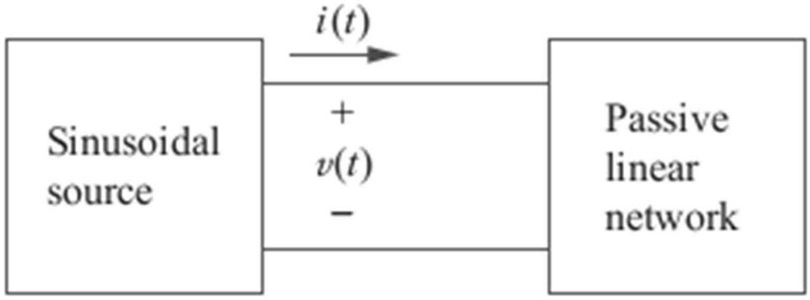
Power may be measured as Instantaneous power or average power.

INSTANTANEOUS & AVERAGE POWER

Power is the rate at which an element absorbs energy.

The Instantaneous Power (watts) is the power at any instant of time.

$$p(t) = v(t)i(t)$$



INSTANTANEOUS & AVERAGE POWER

Consider voltage and current;

$$v(t) = V_m \cos(\omega t + \theta_v)$$

$$i(t) = I_m \cos(\omega t + \theta_i)$$

Instantaneous power;

$$p(t) = v(t)i(t) = V_m I_m \cos(\omega t + \theta_v) \cos(\omega t + \theta_i)$$

Applying trigonometric formula;

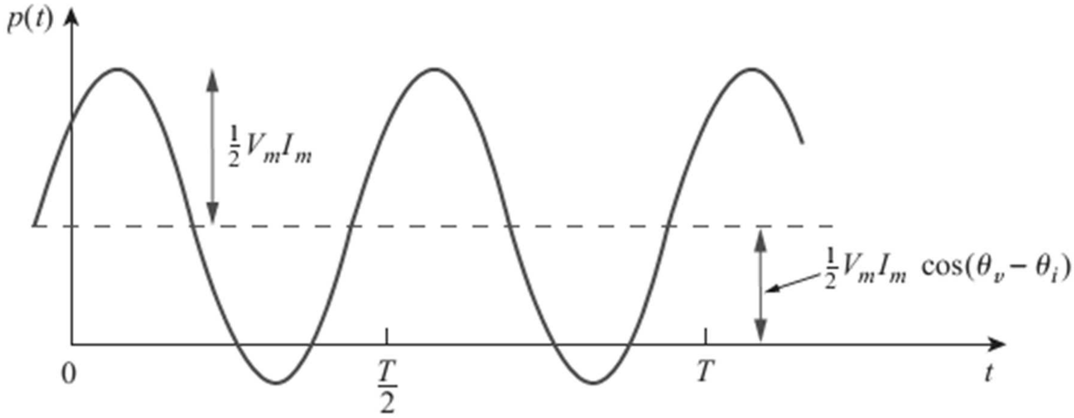
$$\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$$

$$p(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

INSTANTANEOUS & AVERAGE POWER

$$p(t) = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

Instantaneous power has two parts: 1st part is constant and time independent while 2nd part is sinusoidal function with double angular frequency than voltage and current.



INSTANTANEOUS & AVERAGE POWER

From sketch of p(t), it is evident that p(t) is periodic function and has time period of $T_0=T/2$ because its angular frequency is twice than that of voltage or current.

The instantaneous power changes with time and therefore difficult to measure.

The average power is more convenient to measure.

The wattmeter is an electrical instrument which measures average electrical power.

INSTANTANEOUS & AVERAGE POWER

The Average Power, in watts, is the average of the instantaneous power over one period.

$$P = \frac{1}{T}\int_0^T p(t)\,dt$$

Evaluating;

$$P = \frac{1}{T}\int_0^T \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)\,dt + \frac{1}{T}\int_0^T \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)\,dt$$

The 1st integral is constant and average of a constant is the same constant.

$$= \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)\frac{1}{T}\int_0^T dt + \frac{1}{2}V_m I_m \frac{1}{T}\int_0^T \cos(2\omega t + \theta_v + \theta_i)\,dt$$

INSTANTANEOUS & AVERAGE POWER

The 2nd integral is sinusoidal: average value of sinusoidal function is always zero because areas of positive and negative half cycles are equal.

Thus average power will be;

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$$

In phasor form; $\mathbf{V} = V_m\angle\theta_v$ and $\mathbf{I} = I_m\angle\theta_i$.

$$\frac{1}{2}\mathbf{V}\mathbf{I}^* = \frac{1}{2}V_m I_m\angle\theta_v - \theta_i$$

$$= \frac{1}{2}V_m I_m[\cos(\theta_v - \theta_i) + j\sin(\theta_v - \theta_i)]$$

INSTANTANEOUS & AVERAGE POWER

The real part of expression will be average power.

$$P = \frac{1}{2}\text{Re}[\mathbf{VI}^*] = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$$

When $\theta_v = \theta_i$, voltage and current are in phase in case of pure resistive load.

$$P = \frac{1}{2}V_m I_m = \frac{1}{2}I_m^2 R = \frac{1}{2}|\mathbf{I}|^2 R$$

When $\theta_v - \theta_i = \pm 90°$, in case of pure reactive circuit.

$$P = \frac{1}{2}V_m I_m \cos 90° = 0$$

A resistive load absorbs power all the time while reactive load absorbs zero average power.

PROBLEMS

Find instantaneous and average power?

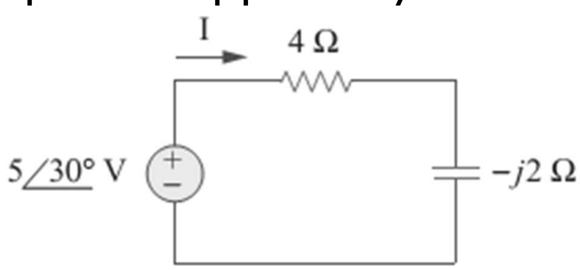
$$v(t) = 120\cos(377t + 45°)\text{ V} \quad i(t) = 10\cos(377t - 10°)\text{ A}$$

$p(t) = 344.2 + 600\cos(754t + 35°)$ W 344.2 W

Find average power?

$\mathbf{Z} = 30 - j70\ \Omega$ $\mathbf{V} = 120\angle 0°$ 37.24 W

Find average power supplied by source and absorbed by resistor?



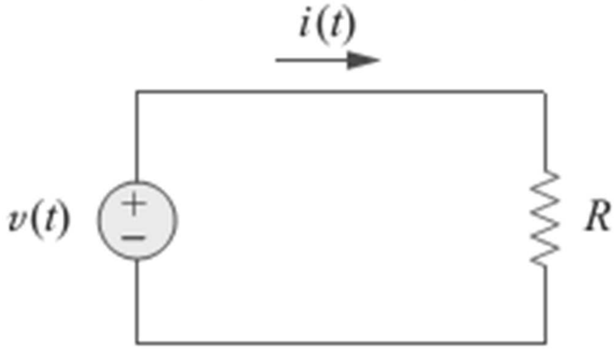
2.5 W

EFFECTIVE OR RMS VALUE

The Effective Value of a periodic current is the dc current that delivers the same average power to a resistor as the periodic current.

The average power absorbed by resistor in ac circuit;

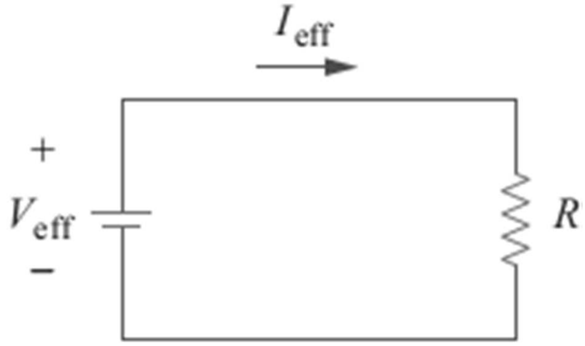
$$P = \frac{1}{T}\int_0^T i^2 R\, dt = \frac{R}{T}\int_0^T i^2\, dt$$



EFFECTIVE OR RMS VALUE

The average power absorbed by resistor in dc circuit;

$$P = I_{\text{eff}}^2 R$$



Equating both powers;

$$I_{\text{eff}} = \sqrt{\frac{1}{T}\int_0^T i^2\, dt}$$

Similarly Voltage;

$$V_{\text{eff}} = \sqrt{\frac{1}{T}\int_0^T v^2\, dt}$$

EFFECTIVE OR RMS VALUE

The Effective Value of a periodic signal is its root mean square (rms) value.

$$I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T I_m^2 \cos^2 \omega t \, dt}$$

$$= \sqrt{\frac{I_m^2}{T}\int_0^T \frac{1}{2}(1 + \cos 2\omega t)\, dt} = \frac{I_m}{\sqrt{2}}$$

Similarly;

$$V_{\text{rms}} = \frac{V_m}{\sqrt{2}}$$

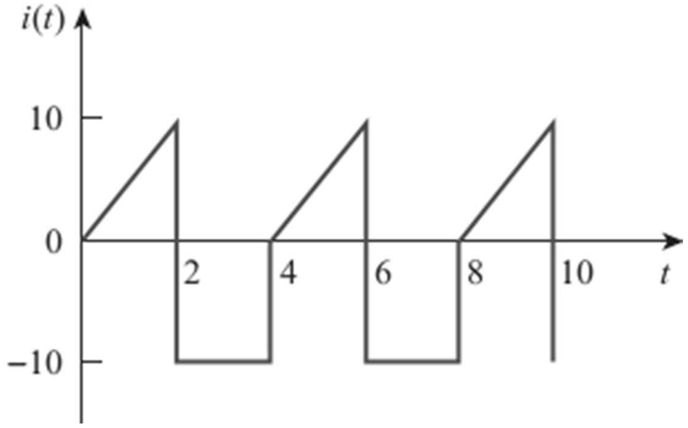
Power in rms;

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos(\theta_v - \theta_i)$$

$$= V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i)$$

PROBLEMS

Determine the rms value of the current waveform? Also fine average power absorbed by 2 Ω resistor?



$$i(t) = \begin{cases} 5t, & 0 < t < 2 \\ -10, & 2 < t < 4 \end{cases}$$

(8.165 A, 133.3 W)

APPARENT POWER AND POWER FACTOR

Sinusoidal voltage and current;

$$v(t) = V_m \cos(\omega t + \theta_v) \qquad i(t) = I_m \cos(\omega t + \theta_i)$$

Phasor voltage and current;

$$\mathbf{V} = V_m \angle \theta_v \qquad \mathbf{I} = I_m \angle \theta_i$$

Average power;

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$P = V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i) = S \cos(\theta_v - \theta_i)$$

$$S = V_{\text{rms}} I_{\text{rms}}$$

APPARENT POWER AND POWER FACTOR

The Apparent Power (in VA) is the product of the rms values of voltage and current.

$$S = V_{\text{rms}} I_{\text{rms}}$$

The Power Factor is the cosine the phase difference between voltage and current.

$$\text{pf} = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

Power factor is the ratio of average power to apparent power that's why this quantity is dimensionless.

APPARENT POWER AND POWER FACTOR

Power factor may also be defined as the cosine of the angle of the load impedance.

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{V_m\underline{/\theta_v}}{I_m\underline{/\theta_i}} = \frac{V_m}{I_m}\underline{/\theta_v - \theta_i}$$

$$\mathbf{V}_{\text{rms}} = \frac{\mathbf{V}}{\sqrt{2}} = V_{\text{rms}}\underline{/\theta_v} \qquad \mathbf{I}_{\text{rms}} = \frac{\mathbf{I}}{\sqrt{2}} = I_{\text{rms}}\underline{/\theta_i}$$

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{\mathbf{V}_{\text{rms}}}{\mathbf{I}_{\text{rms}}} = \frac{V_{\text{rms}}}{I_{\text{rms}}}\underline{/\theta_v - \theta_i}$$

APPARENT POWER AND POWER FACTOR

The value of power factor (pf) ranges between zero and unity.

For pure resistive load, $\theta_v - \theta_i = 0$ $\quad$ $\text{pf} = 1$

For pure reactive load, $\theta_v - \theta_i = \pm 90^\circ$ $\quad$ $\text{pf} = 0.$

In between these two extreme cases, pf is said to be leading or lagging.

In capacitive load, current leads voltage which causes power factor to be leading.

In inductive load, current lags voltage which causes power factor to be lagging.

PROBLEMS

Find apparent power and power factor?

$$i(t) = 4\cos(100\pi t + 10°)\text{ A}$$
$$v(t) = 120\cos(100\pi t - 20°)\text{ V}$$

(240 VA, 0.866 lag)

COMPLEX POWER

The apparent power represented in phasor form is known as complex power.

Consider;

$$\mathbf{S} = \frac{1}{2}\mathbf{V}\mathbf{I}^*$$

$$\mathbf{S} = \mathbf{V}_{\text{rms}}\mathbf{I}^*_{\text{rms}}$$

$$\mathbf{V}_{\text{rms}} = \frac{\mathbf{V}}{\sqrt{2}} = V_{\text{rms}}\underline{/\theta_v} \qquad \mathbf{I}_{\text{rms}} = \frac{\mathbf{I}}{\sqrt{2}} = I_{\text{rms}}\underline{/\theta_i}$$

$$\begin{aligned}\mathbf{S} &= V_{\text{rms}}I_{\text{rms}}\underline{/\theta_v - \theta_i} \\ &= V_{\text{rms}}I_{\text{rms}}\cos(\theta_v - \theta_i) + jV_{\text{rms}}I_{\text{rms}}\sin(\theta_v - \theta_i)\end{aligned}$$

COMPLEX POWER

Real parts of complex power is called Real Power;

$$P = V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i).$$

Imaginary part of complex power is called Reactive Power;

$$Q = V_{\text{rms}} I_{\text{rms}} \sin(\theta_v - \theta_i)$$

Complex Power (VA) is the product of rms voltage phasor and complex conjugate of the rms current phasor. As a complex quantity, its real part is real power P (Watts) and imaginary part is reactive power Q (VARS).

COMPLEX POWER

$$\text{Complex Power} = \mathbf{S} = P + jQ = \mathbf{V}_{\text{rms}}(\mathbf{I}_{\text{rms}})^*$$

$$= \mathbf{V}_{\text{rms}}\mathbf{I}_{\text{rms}}\underline{/\theta_v - \theta_i}$$

$$\text{Apparent Power} = S = |\mathbf{S}| = \mathbf{V}_{\text{rms}}\mathbf{I}_{\text{rms}} = \sqrt{P^2 + Q^2}$$

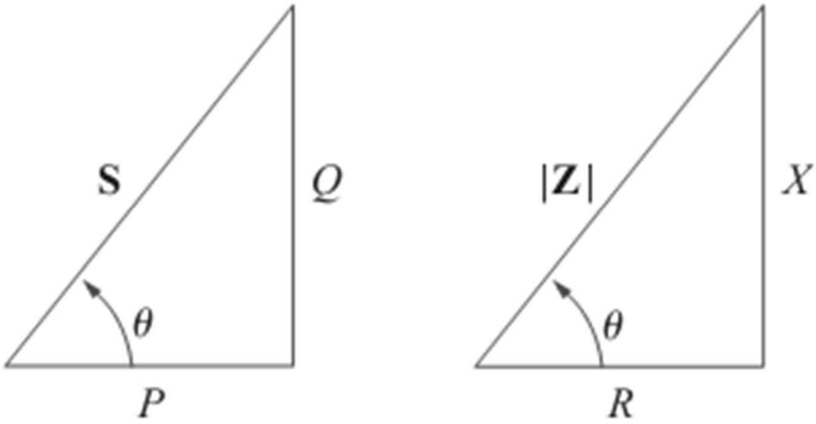
$$\text{Real Power} = P = \text{Re}(\mathbf{S}) = S\cos(\theta_v - \theta_i)$$

$$\text{Reactive Power} = Q = \text{Im}(\mathbf{S}) = S\sin(\theta_v - \theta_i)$$

$$\text{Power Factor} = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

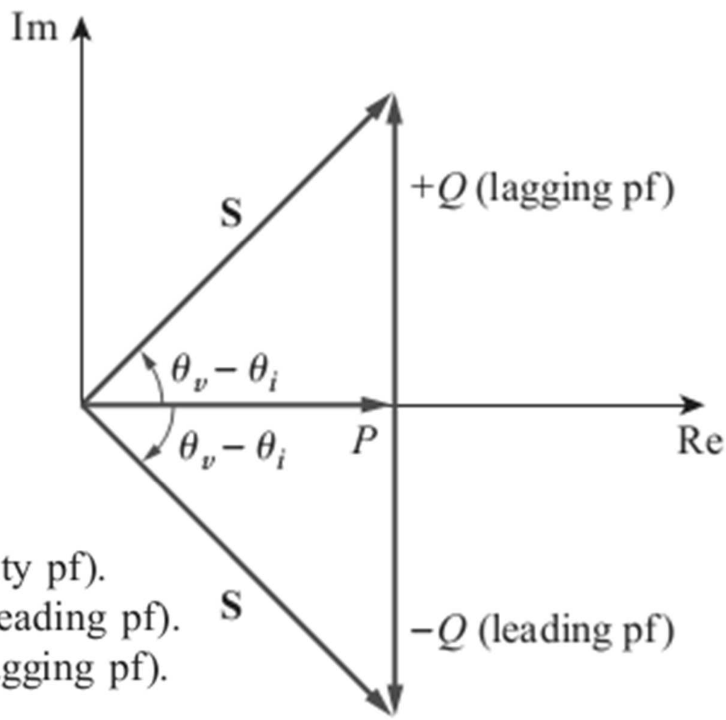
COMPLEX POWER

Power Triangle;



COMPLEX POWER

Power Triangle;



1. $Q = 0$ for resistive loads (unity pf).
2. $Q < 0$ for capacitive loads (leading pf).
3. $Q > 0$ for inductive loads (lagging pf).

PROBLEMS

Find complex power, apparent power, real power, reactive power, power factor and load impedance?

$$v(t) = 60\cos(\omega t - 10°)\text{ V}$$

$$i(t) = 1.5\cos(\omega t + 50°)\text{ A}$$

CONSERVATION OF AC POWER

The principle of conservation of power applies to ac circuits as well as to dc circuits.

The complex, real and reactive powers of the sources equal the respective sums of the complex, real and reactive powers of the individual loads.

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 + \cdots + \mathbf{S}_N$$

REFERENCES

Fundamentals of Electric Circuits (4th Edition)

Charles K. Alexander, Matthew N. O. Sadiku

Chapter 11 – AC Power Analysis

(11.1 – 11.2, 11.4 – 11.7)

Exercise Problems: 11.1 – 11.11, 11.22 – 11.68

Do exercise problem yourself.